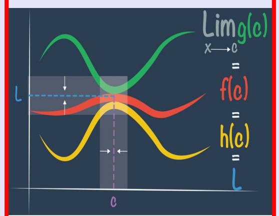
Calculus I

Lecture 52



Feb 19-8:47 AM

Find f_{ave} for $f(x) = \cos x$ over $[0, 2\pi]$.

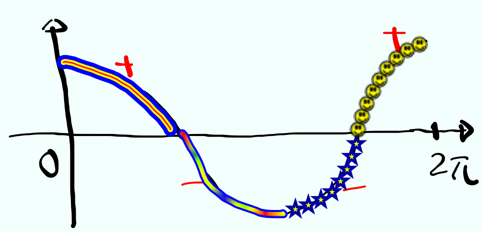
Cont. & diff.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{2\pi - 0}\int_0^{2\pi} \cos x\,dx = \frac{1}{2\pi}\left[\sin x\right]\Big|_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\cancel{\sin 2\pi}^{\,0} - \cancel{\sin 0}^{\,0}\right] = \boxed{0}$$

$f(x) = \cos x$



Dec 5-7:26 AM

find f_{ave} for $f(x) = \underbrace{\sec^2 \frac{x}{2}}_{\text{cont.}}$ on $\left[0, \frac{\pi}{2}\right]$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{\frac{\pi}{2} - 0}\int_0^{\pi/2} \sec^2 \frac{x}{2}\,dx$$

$$= \frac{1}{\frac{\pi}{2}}\int_0^{\pi/2} \sec^2 \frac{x}{2}\,dx = \frac{2}{\pi}\int_0^{\pi/2} \sec^2 \frac{x}{2}\,dx$$

$u = \frac{x}{2}$

$2u = x$

$2du = dx$

$x = 0 \rightarrow u = \frac{0}{2} = 0$

$x = \frac{\pi}{2} \rightarrow u = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

$$= \frac{2}{\pi}\int_0^{\pi/4} \sec^2 u \cdot 2du$$

$$= \frac{4}{\pi}\int_0^{\pi/4} \sec^2 u\,du$$

$$= \frac{4}{\pi} \tan u \Big|_0^{\pi/4}$$

$$= \frac{4}{\pi}\left[\cancelto{1}{\tan\frac{\pi}{4}} - \cancelto{0}{\tan 0}\right] = \boxed{\frac{4}{\pi}}$$

Dec 5-7:32 AM

find f_{ave} for $f(x) = \underbrace{\frac{x}{1+x^2}}_{\text{Cont. } (-\infty,\infty)}$ over $[0,2]$.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$x = 0 \rightarrow u = 1$

$x = 2 \rightarrow u = 5$

$$= \frac{1}{2-0}\int_0^2 \frac{x}{1+x^2}\,dx = \frac{1}{2}\int_0^2 \frac{x}{1+x^2}\,dx$$

$u = 1 + x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$= \frac{1}{2}\int_1^5 \frac{1}{u}\,\frac{du}{2}$$

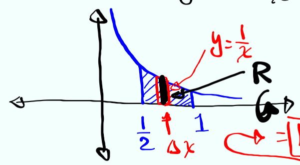
$$= \frac{1}{4}\int_1^5 \frac{1}{u}\,du$$

$$= \frac{1}{4} \ln u \Big|_1^5 = \frac{1}{4}\left[\ln 5 - \cancelto{0}{\ln 1}\right] = \boxed{\frac{\ln 5}{4}}$$

Dec 5-7:39 AM

Consider the region bounded by $y=\frac{1}{x}$, $y=0$, $x=\frac{1}{2}$ and $x=1$.

1) Draw it



2) Find its area

$$A=\int_{1/2}^{1}\frac{1}{x}\,dx=\ln x\Big|_{1/2}^{1}=\ln 1-\ln\tfrac{1}{2}=-(\ln 1-\ln 2)=\boxed{\ln 2}$$

3) Find the volume if rotated about x-axis.

Disk Method

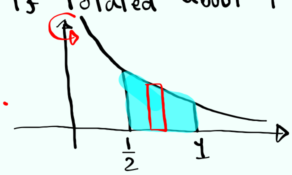
$$V=\int_{1/2}^{1}\pi\left[\frac{1}{x}\right]^2dx=\pi\int_{1/2}^{1}\frac{1}{x^2}dx=\pi\int_{1/2}^{1}x^{-2}dx=\pi\cdot\frac{x^{-1}}{-1}=-\pi\cdot\frac{1}{x}\Big|_{1/2}^{1}=\boxed{\pi}$$

$$-\pi\left[\frac{1}{1}-\frac{1}{\frac{1}{2}}\right]=-\pi[1-2]=-\pi\cdot(-1)=\pi$$

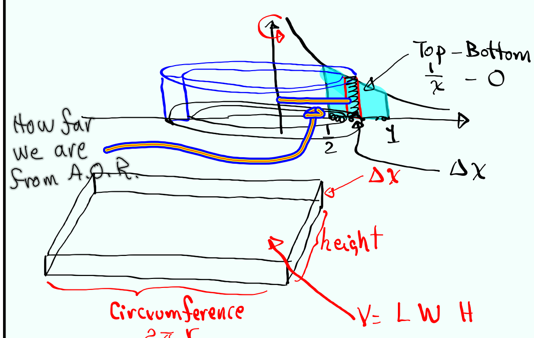
4) Find the volume if rotated about y-axis.

Since Ref. Rect. is Parallel to the A.O.R.

⇒ Shell Method



Dec 4-8:10 AM



$V = LWH$

$V=2\pi r\cdot \text{height}\ \Delta x$

Shell Method

$$V=\int_a^b 2\pi\, D\, H\, dx$$

D: How far from A.O.R.

H: Height of Ref. Rect.

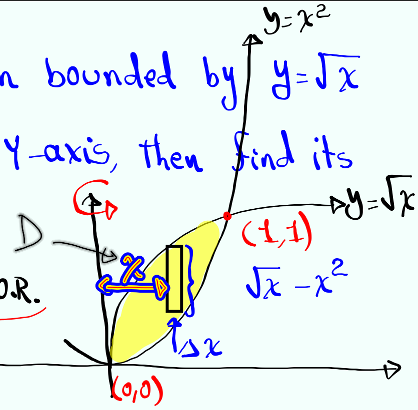
$$V=\int_{1/2}^{1}2\pi\,\cancel{x}\,\frac{1}{\cancel{x}}\,dx=2\pi\int_{1/2}^{1}dx=2\pi x\Big|_{1/2}^{1}$$

$$=2\pi\left[1-\frac{1}{2}\right]$$

$$=2\pi\cdot\frac{1}{2}=\boxed{\pi}$$

Dec 5-7:50 AM

Rotate the region bounded by $y=\sqrt{x}$ and $y=x^2$ by Y-axis, then find its Volume.



Ref. Rect. parallel to A.O.R.

Shell Method

$$V=\int_0^1 2\pi x\left(\sqrt{x}-x^2\right)dx$$

$$=2\pi\int_0^1\left(x^{3/2}-x^3\right)dx=2\pi\left[\frac{x^{5/2}}{5/2}-\frac{x^4}{4}\right]\Bigg|_0^1$$

$$=2\pi\left[\frac{2}{5}x^{5/2}-\frac{1}{4}x^4\right]\Bigg|_0^1=2\pi\left[\frac{2}{5}-\frac{1}{4}\right]$$

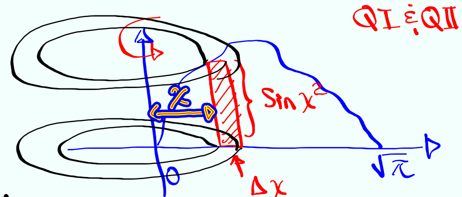
$$=2\pi\cdot\frac{3}{20}=\frac{3\pi}{10}$$

Dec 5-7:59 AM

Cont.

Rotate the region bounded by $f(x)=\sin x^2$, $g(x)=0$ on $[0,\sqrt{\pi}]$ about Y-xis.

+ in QI & QII



Shell Method

$$V=\int_0^{\sqrt{\pi}} 2\pi \underset{D}{x}\,\underset{H}{\sin x^2}\,dx$$

$u=x^2$

$du=2x\,dx$

$x=0 \quad u=0$

$x=\sqrt{\pi} \quad u=\pi$

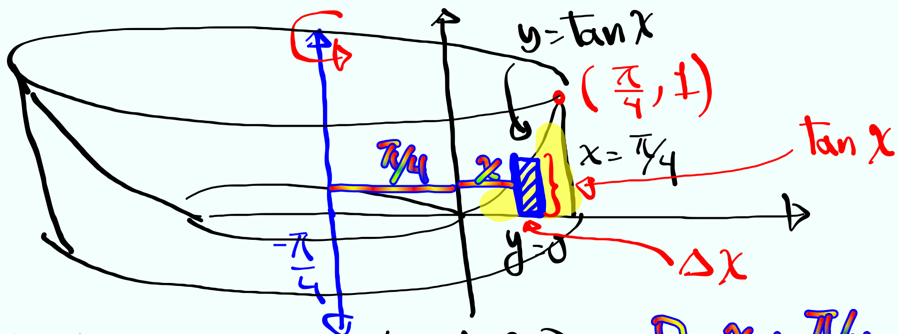
$$=\pi\int_0^{\pi}\sin u\,du$$

$$=\pi\left[-\cos u\right]\Big|_0^{\pi}=-\pi\left(\overset{-1}{\cos\pi}-\overset{1}{\cos 0}\right)$$

$$=-\pi(-1-1)=\boxed{2\pi}$$

Dec 5-8:07 AM

Rotate the region bounded by $y = \tan x$, $y = 0$, $x = \frac{\pi}{4}$ about $x = -\frac{\pi}{4}$. Find its Volume.



Ref. Rect. is parallel to A.O.R.

$D = x + \pi/4$

$H = \tan x$

Shell Method

$$V = \int_0^{\pi/4} 2\pi\left(x + \frac{\pi}{4}\right) \cdot \tan x \, dx.$$

wait for Cal. II.

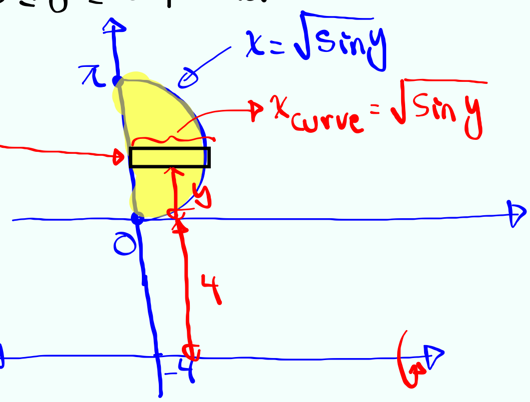
Dec 5-8:15 AM

Rotate the region bounded by

$x = \sqrt{\sin y}$, $0 \le y \le \pi$, and $x = 0$

about $y = -4$.

Δy



Ref. Rect. parallel to A.O.R.

Shell Method

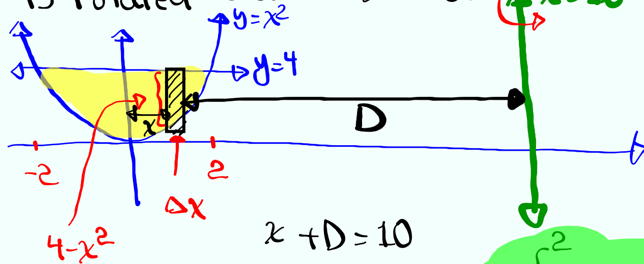
$H = \sqrt{\sin y}$

$D = y + 4$

$$V = \int_0^{\pi} 2\pi\,(y+4)\sqrt{\sin y}\; dy$$

Dec 5-8:23 AM

Set-up the integral for the volume when the region bounded by $y=x^2$, and $y=4$ is rotated about $x=10$.



$$x + D = 10$$
$$D = 10 - x$$

$$V = \int_{-2}^{2} 2\pi(10-x)(4-x^2)\,dx$$

$$= 2\pi\int_{-2}^{2}\left[40 - 10x^2 - 4x + x^3\right]dx$$

Dec 5-8:29 AM